MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2022/2023**

## BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

13 JULY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

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**INSTRUCTIONS TO STUDENTS**

1. This question paper consists of **SIX (6)** printed pages inclusive of the cover page, formulae sheet.
2. Answer **ALL** four questions in answer booklet provided.
3. Students are allowed to use authorized calculators by lecturer only.
4. Marks are shown at the end of each question.

**Question 1 [20 marks]**

(a) A basketball player gets 6 independent free throws. The probability of getting a basket on each shot is 0.8, what is the probability that he/she gets exactly 4 baskets? Calculate the mean and standard deviation of getting a basket on each shot. [4 marks]

(b) The price of a stock is uniformly distributed between RM20 and RM40.

(i) What is the probability that the stock price will be more than RM30? [4 marks]

(ii) Determine the expected and standard deviation of the stock price. [4 marks]

(c) The time taken by a wife to shop for anniversary cards follows an exponential distribution, with an average duration of eight minutes.

(i) What will be the probability that she will take more than 10 minutes to shop for anniversary cards? [4 marks]

(ii) What will be the probability that she will take from 7 to 9 minutes to shop for anniversary cards? [4 marks]

**Question 2 [30 marks]**

(a) Suppose that X is a random variable that has the following probability function:

$$f(x) = 3e^{-x} \quad x > 0$$

Use the moment generating function (MGF) to find the E(X) and the $E(X^2)$. Next calculate the standard deviation. [18 marks]

(b) Given the joint probability density function:

$$f(x,y) = \begin{cases} 4xy & 0 \le x \le 1; 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

(i) Find the marginal pdf of X and Y. [6 marks]

(ii) Find $P\left(Y \le \frac{1}{2}, X \le \frac{1}{2}\right)$. [6 marks]

**Question 3 [20 marks]**

In a monthly staff meeting, staff reached the meeting room with an average of 2 staff per minute. A member of staff has just reached the meeting room.

(i) Find the probability that the interarrival time for the next staff to reach the meeting room is less than 5 seconds. [5 marks]

**Continued...**

(ii) Find the probability that the chair meeting needs to wait more than 10 seconds for the next staff to arrive. [4 marks]

(b) In a simple random walk, the probability that the walk advances by one step is $p = 0.56$ and retreats by one step is q = 0.44. Consider the starting point, $X_0 = 0$.

(i) find the probability that the walker is at the position +3 after 5 steps. [6 marks]

(ii) If the walker has walked for 10 steps, find the mean and standard deviation. [5 marks]

**Question 4 [30 marks]**

A company is considering using Markov theory to analyse brand switching between three different brands of hard disks (Brand A, B and C). Survey data has been gathered and has been used to estimate the following transition matrix for the probability of moving between brands each month:

$$\mathbf{P} = \begin{bmatrix} 0.80 & 0.10 & 0.10 \\ 0.30 & 0.95 & 0.02 \\ 0.20 & 0.05 & 0.75 \end{bmatrix}$$

[Note: Leave your answer in 2 decimal places]

(a) Draw the transition diagram. [4 marks]

(b) If a person is currently a brand B purchaser, what is the probability that he will purchase brand C two months from now? [5 marks]

(c) Given initially the share for Brand A, Brand B and Brand C is 20%, 40% and 40% respectively. Find the share of Brand C after 2 years. [5 marks]

(d) Determine the stationary distribution. [13 marks]

(e) Determine whether all the states are absorbing or non-absorbing states. [3 marks]

**End of Questions**

# FORMULAE

## A. PROBABILITY DISTRIBUTION

**Bernoulli Probability Distribution**

$P(X = x) = p^x q^{1-x}$ for $x = 0, 1$

**Binomial Probability Distribution**

$P(X = x) = {}^nC_x \, p^x q^{n-x}$ for $x = 0, 1, \ldots, n$

E(X) = np ; Var (X) = np (1 – q)

**Poisson Probability Distribution**

$P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, 2, \ldots$

E(X) = Var (X) = $\lambda$

**Geometric Probability Distribution**

$P(X = x) = pq^{x-1}$ for $x = 1, 2, \ldots$

E(X) = $\dfrac{1}{p}$ ; Var (X) = $\dfrac{1-p}{p^2}$

**Uniform Probability Distribution**

$f(x) = \dfrac{1}{b-a}$ for $a < x < b$

E(X) = $\dfrac{a+b}{2}$ ; Var (X) = $\dfrac{(b-a)^2}{12}$

**Exponential Probability Distribution**

$f(x) = \lambda e^{-\lambda x}$ for $x > 0$

E(X) = Var (X) = $\dfrac{1}{\lambda}$

**Continued....**

## B. MOMENT

**$k^{th}$ Moment about the origin of X:**

$$\mu'_k = E[X^k]$$

**Moment Generating Function for X:**

$$M_X(t) = E\left(e^{tx}\right)$$

$$E(X) = M_x^{(1)}(t=0) \quad ; \quad E(X^2) = M_x^{(2)}(t=0)$$

## C. STOCHASTIC PROCESS

**Simple Random Walk:**

$$P(X_n = m) = \binom{n}{\frac{n+m}{2}} p^{\left(\frac{n+m}{2}\right)} q^{\left(\frac{n-m}{2}\right)}$$ where $m$ and $n$ are both even or both odd.

$E(X_n) = n(p - q)$ ; $Var(X_n) = 4npq$

**Gambler's Ruin Problem**

$$P_a = \begin{cases} \dfrac{\left(\frac{q}{p}\right)^a - \left(\frac{q}{p}\right)^c}{1 - \left(\frac{q}{p}\right)^c}, & p \neq q \\ 1 - \dfrac{a}{c}, & p = q \end{cases} \quad ; \quad P_a + P_b = 1$$

$$D_a = \begin{cases} \dfrac{1}{q-p}\left\{a - c\left[\dfrac{1-\left(\frac{q}{p}\right)^a}{1-\left(\frac{q}{p}\right)^c}\right]\right\}, & p \neq q \\ a(c-a), & p = q \end{cases}$$

**Continued....**

## D. POISSON PROCESS

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| **Pmf for N(t) :** $$P\{N(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$$ |
| **Pdf for $T_n$:** $$f(t)=\lambda e^{-\lambda t}$$ |
| **Pdf for $S_n$:** $$f(t)=\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$ |

**End of Pages.**